



SE-8047

B. E. II (Sem. III) (All) Examination

May / June - 2011

Engineering Mathematics - III

Time : 3 Hours]

[Total Marks : 100

Instructions :

(1)

नीचे दर्शाविए निशानीवाणी विगतो उत्तरवही पर अवश्य लपवी.  
Fillup strictly the details of signs on your answer book.

Name of the Examination :  
**B. E. 2 (Sem. 3) (All)**

Name of the Subject :  
**Engineering Mathematics - 3**

Subject Code No. : **8 0 4 7** Section No. (1, 2,.....) : **Nil**

Seat No. :

Student's Signature

- 1) All questions are compulsory.
- 2) Figures on the right indicate marks
- 3) Draw the figure whenever it is necessary.

1 a) Attempt the following [10]

1. Find the value of  $\lambda$  for which the equation  $(xy^2 + \lambda x^2 y)dx + (x + y)dy = 0$  is exact.
2. Use method of reduction to find the second L.I solution of  $y'' + 6y' + 9y = 0$  given that  $y_1(x) = e^{-3x}$  is one solution.
3. Find the complementary function of  $y'' + 2y' + y = e^x$ .
4. State orthogonality property for Legendre polynomials.
5. Evaluate  $\int_{-1}^1 P_2^2(x) dx$

b) Attempt any two of the following [10]

1. Find power series solution of  $y'' + y = 0$
2. Obtain a Frobenius series solution of  $4xy'' + 2(1-x)y' - y = 0$
3. Use Frobenius series solution method to obtain the solution of  $x^2 y'' + xy' + (x^2 - 4)y = 0$

2 a) Attempt any two of the following [6]

1. Solve  $xdy - ydx = \sqrt{x^2 + y^2}$
2. Solve  $\frac{dy}{dx} + \frac{x}{1-x^2}y = x\sqrt{y}$ .
3. Solve  $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$ .

b) Attempt any three of the following [9]

1. Solve  $y'' + 2y' + 3y = 20 \cos 2x$ .
2.  $x^2 y'' - 4xy' + 6y = 21x^{-4}$ .
3. Solve using Method of variation of parameters,  $y'' + y = \sec x$ .
4. Use method of undetermined coefficient to solve  $y'' + 4y = 8x^2$

3 a) Attempt any three of the following. [9]

1.  $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$ .
2. Prove that  $\int_0^{\infty} e^{-ax^n} dx = \frac{1}{n\sqrt[n]{a}} \Gamma\left(\frac{1}{n}\right)$ .
3. Examine whether  $\sqrt{x}, \sqrt{x} \ln x, \sqrt{x} (\ln x)^2, \dots, \sqrt{x} (\ln x)^k$  are linearly independent.
4. Define Rodrigue's formula & hence derive  $P_3(x)$  from it.

b) Attempt any two of the following. [6]

1. Find Fourier Sine integral of  $f(x) = \begin{cases} \sin x; & 0 \leq x \leq \pi \\ 0 & ; x > \pi \end{cases}$
2. Find the Fourier transform of  $f(x) = xe^{-x^2}$ .
3. Find Fourier cosine transform of  $f(x) = \begin{cases} k & \text{if } 0 < x < a \\ 0 & \text{if } x > a \end{cases}$ .

4 a) Attempt the following. [10]

1. State second shifting theorem & using it evaluate,  $L[4 \sin(t-3)u(t-3)]$
2. Find  $L^{-1}\left(\frac{1}{2s-3}\right)$ .
3. State one dimensional heat equation & state which method we apply to find the solution of it.
4. Define Fourier Integral in the complex form and hence define Fourier transform.
5. Define finite Fourier sine and cosine transforms.

b) Solve any two following using Laplace transform technique. [10]

1.  $y'' - 3y' + 2y = 4t + e^{3t}$ ,  $y(0) = 1$ ,  $y'(0) = -1$ .
2.  $y'' + 2y' + 5y = e^{-t} \sin t$ ,  $y(0) = 0$  &  $y'(0) = 1$ .
3.  $y'' - 2y' - 8y = 0$ ,  $y(0) = 3$ ,  $y'(0) = 6$

5 a) Attempt any two of the following. [10]

1. Obtain the Fourier series to represent  $f(x) = \frac{1}{4}(\pi - x)^2$ ,  $0 < x < 2\pi$ .

if  $f(x) = \begin{cases} 0, & -\pi \leq x \leq 0 \\ \sin x, & 0 \leq x \leq \pi \end{cases}$  then prove that

2.  $f(x) = \frac{1}{\pi} + \frac{1}{2} \sin x - \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{\cos 2nx}{4n^2 - 1}$ .

3. Obtain Fourier series expansion for  $f(x) = \frac{1}{2}(\pi - x)$  for  $0 < x < 2$

b) Attempt any two of the following. [6]

1. Find Laplace transform of  $\sin 2t \cos 3t$ .

2. Find Laplace transform of  $e^{-3t} (\cos 4t + 3 \sin 4t)$ .

3. Find  $L^{-1} \left[ \frac{1}{s^3(s+1)} \right]$

6 a) Solve the following One-dimensional Heat equation [7]

$$u_t = c^2 u_{xx}; t > 0 \text{ and } 0 < x < l$$

$$u(0, t) = 0 = u(l, t); t > 0$$

$$u_t(x, 0) = f(x)$$

b) Attempt any one of the following. [7]

1. Find the deflection  $u(x, t)$  of the vibrating string of length  $\pi$  and its ends are fixed, corresponding to zero initial velocity and the initial deflection is  $u(x, 0) = 2(\sin x + \sin 3x)$ .

2. Solve the boundary value problem,  $u_{xx} + u_{yy} = 0$  conditions are,

$$u(x, 0) = u(0, y) = u(l, y) = 0 \text{ \&}$$

$$u(x, 0) = \frac{\sin n\pi x}{l}; 0 \leq x \leq l \text{ \& } 0 \leq y \leq a$$